Temperate families

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University of Passau

based on joint work with Pavel Turek

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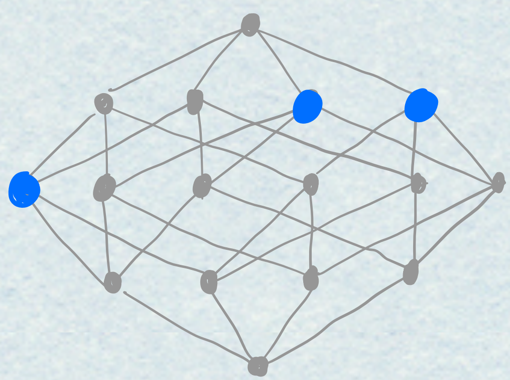

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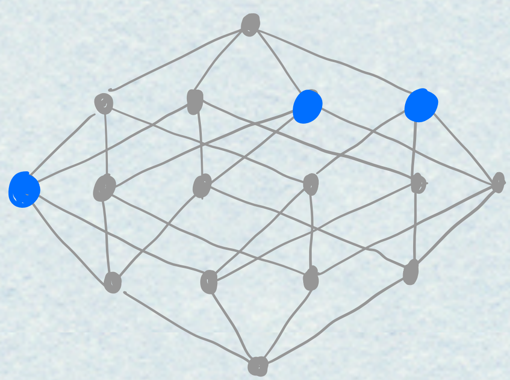

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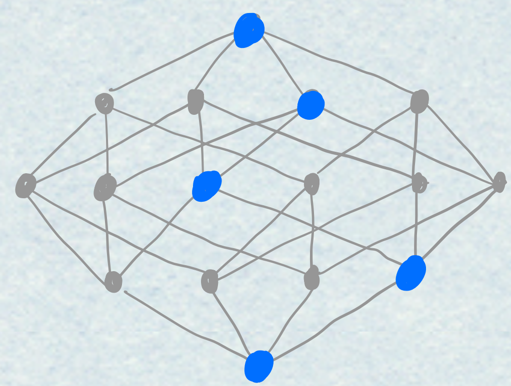

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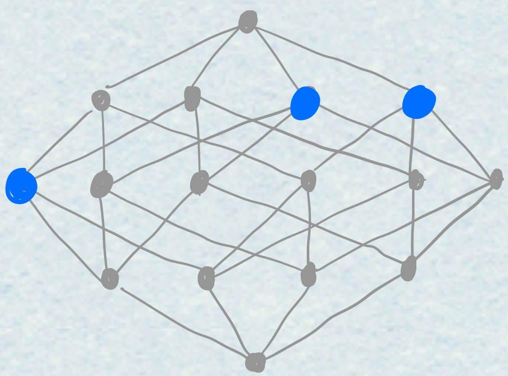

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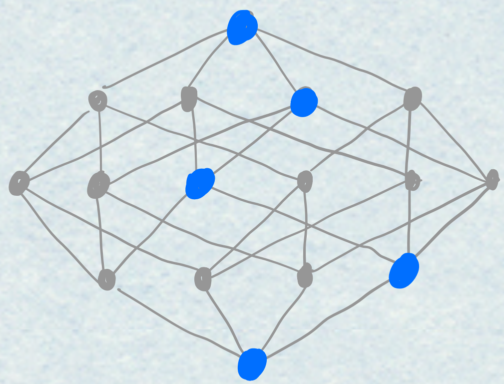
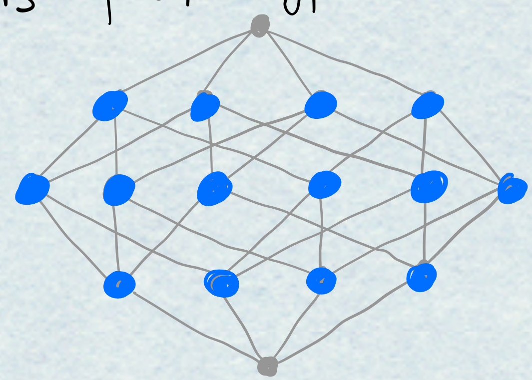


union of $s+1$ consecutive layers of the hypercube $(t \geq s)$



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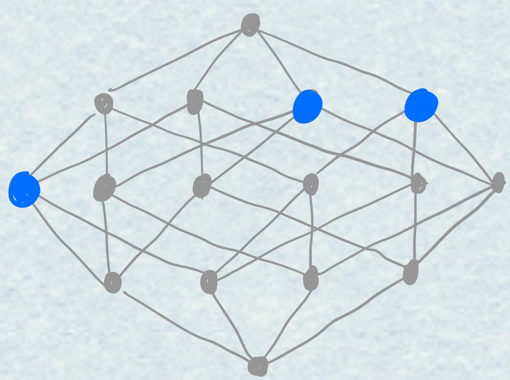

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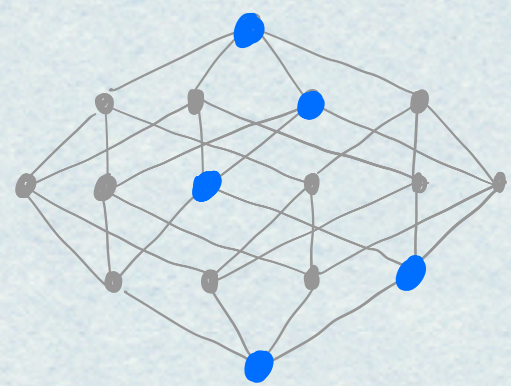

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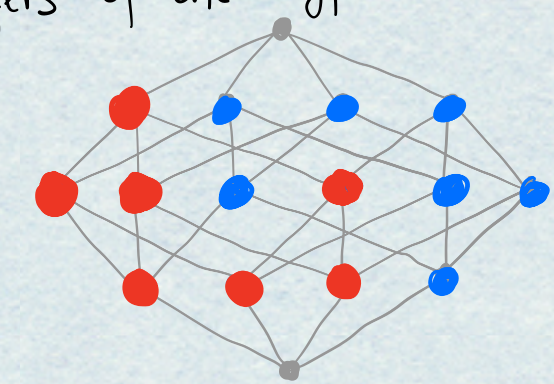

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Question 2. What is the maximum size of an intersecting t-temperate family on $[n]$?

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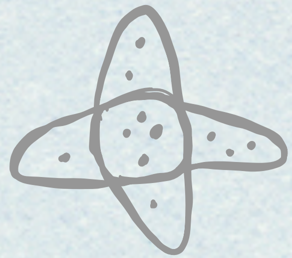

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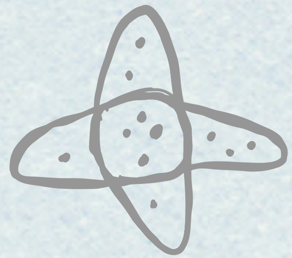

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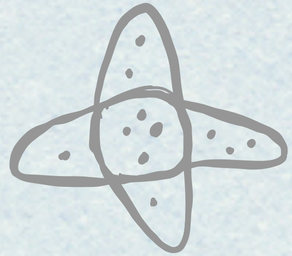

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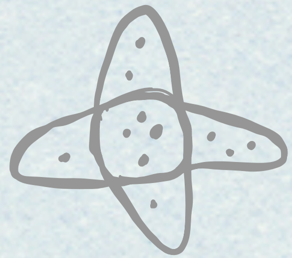

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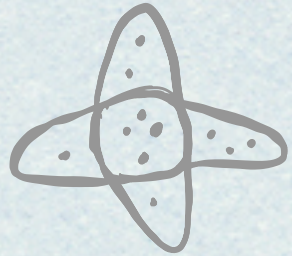

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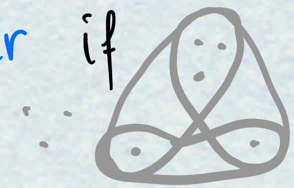

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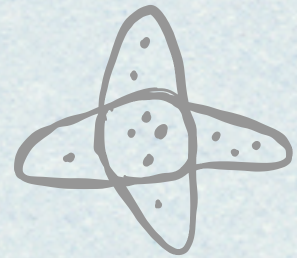

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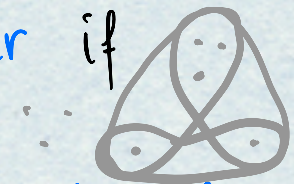

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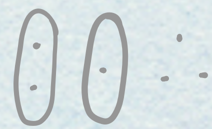

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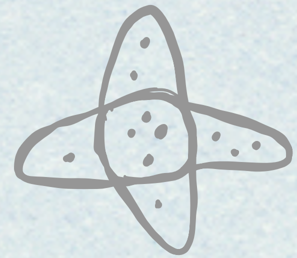

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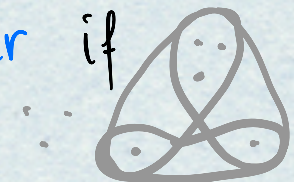

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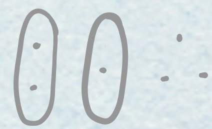

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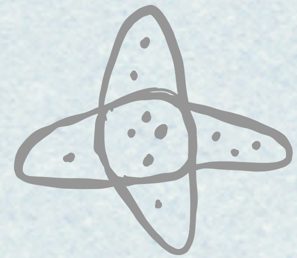

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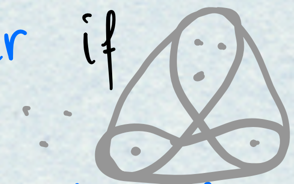

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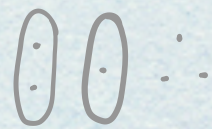

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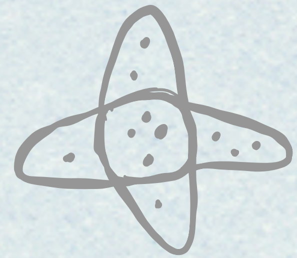

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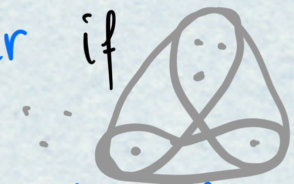

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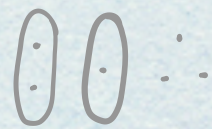

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"Odd-town" => $f_{even}(n) = n$. Frankl, Pach, Pálvölgyi, 2024: $\lim_{n \to \infty} f_{odd}(n)^{1/n} > 1.502$.

$$\lim_{n \to \infty} f_{odd}(n)^{1/n} \leq \phi$$

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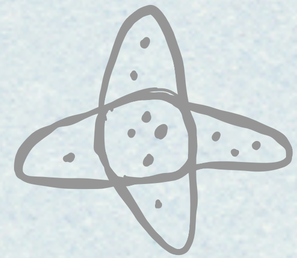

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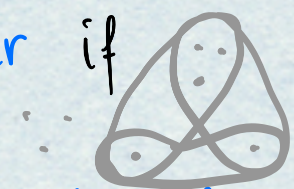

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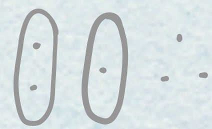

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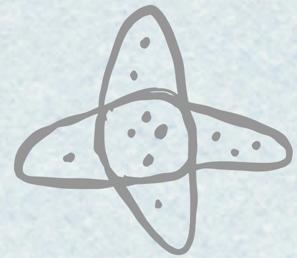

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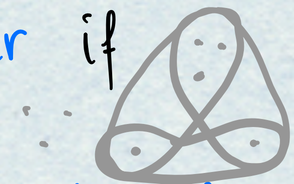

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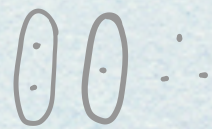


Let $f_{even}(n)$ be the maximum size of an even-sunflower-free family on $[n]$.

$f_{odd}(n)$ odd-sunflower-free

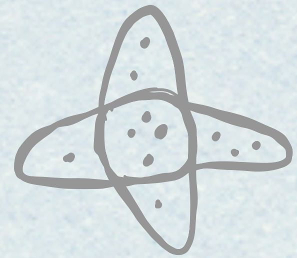
"Odd-town" $\Rightarrow$ $f_{even}(n) = n$. Frankl, Pach, Pálvölgyi, 2024: $\lim_{n\to\infty} f_{odd}(n)^{1/n} > 1.502$.

$\geq \phi$

Observations: (1) Odd-sunflower-free families are temperate.

(2)

Motivation



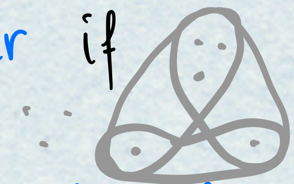
A family of $r \geq 3$ sets $S_1, \dots, S_r$ is an r-sunflower if $\forall i \neq j \quad S_i \cap S_j = \bigcap_{k=1}^{r} S_k$. A sunflower is a family which is an r-sunflower for some $r \geq 3$.

Let $f(n)$ be the maximum size of a sunflower-free family on $[n]$.

Q: What is the value of $\phi = \lim_{n\to\infty} f(n)^{1/n}$? (Note: the limit exists)

Deuber, Erdős, Gunderson, Kostochka, Meyer, 1997: $\phi > 1.551$

Naslund, Sawin, 2017: $\phi < 1.890$.



A non-empty family on $[n]$ of non-empty sets is an even-sunflower if each $i \in [n]$ lies in an even number of sets.

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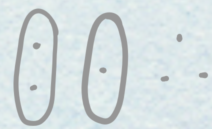

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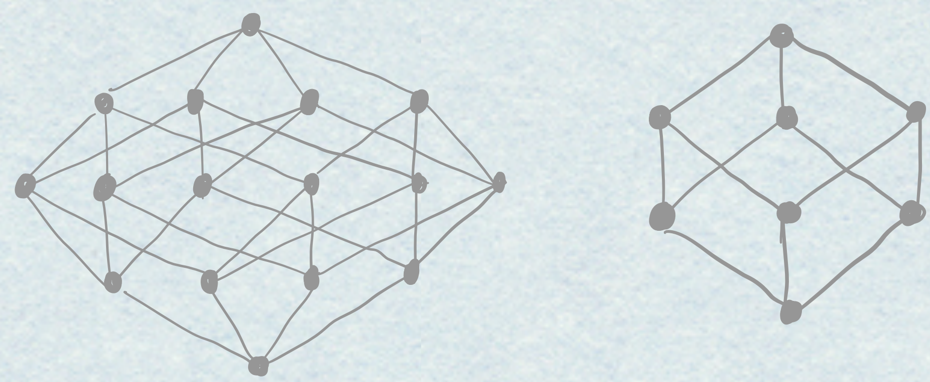
$\rotatebox{90}{$\leq$}\ \phi$

Observations: (1) Odd-sunflower-free families are temperate.
(2) Odd-sunflower-free families are intersecting.

Answer to Question 1



Answer to Question 1

Thm 1: Let $t \le n$ be non-negative integers.
The maximum size of a t-temperate family on $[n]$
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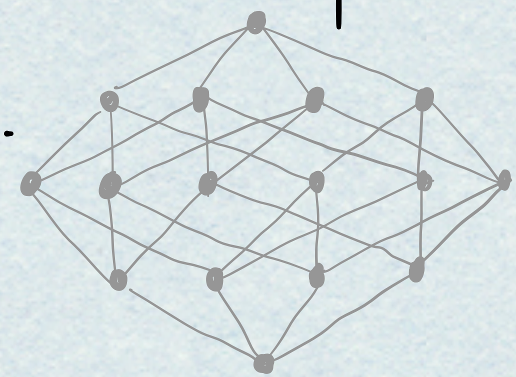

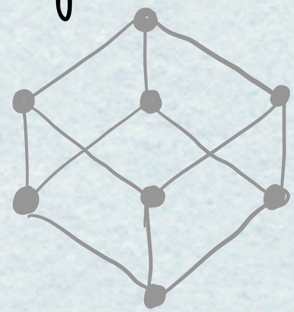

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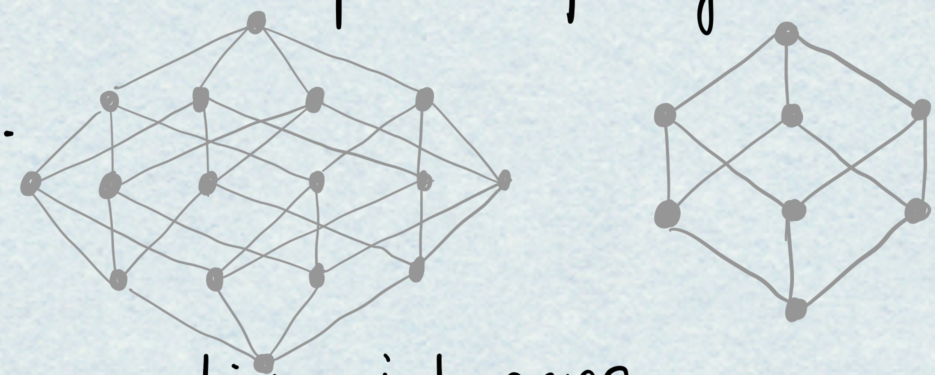

Thm 2: Let $t \le n$ be non-negative integers.

(a)

(b)

(c)

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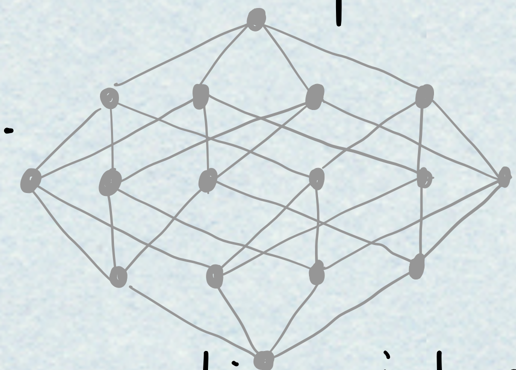

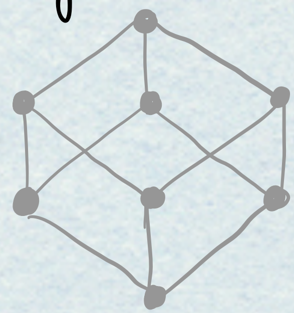

Thm 2: Let $t \leq n$ be non-negative integers.

(a) If $t \equiv n \pmod 2$, the only t-temperate family of maximal size is $\bigcup_{j=0}^{t} \binom{[n]}{\frac{n-t}{2} + j}$.

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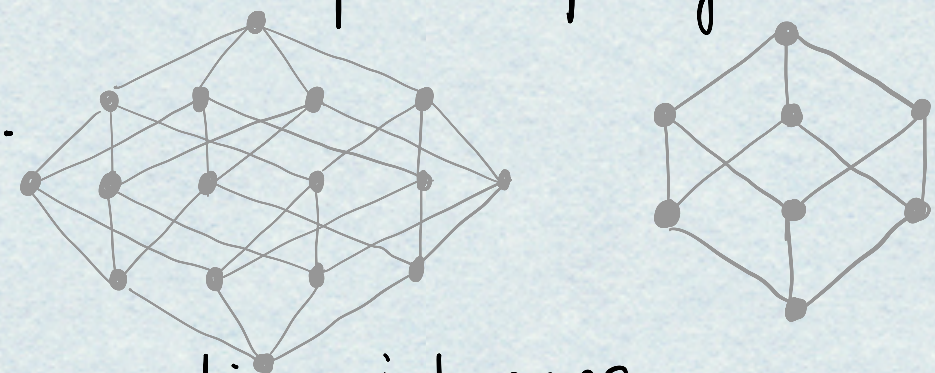

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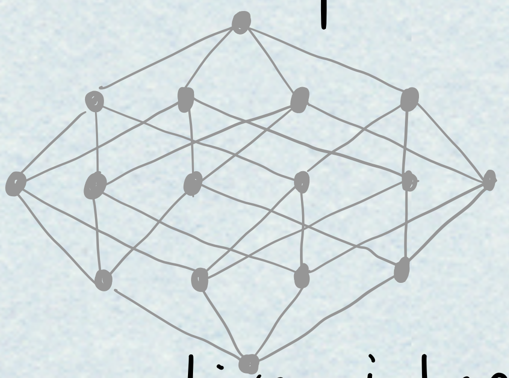

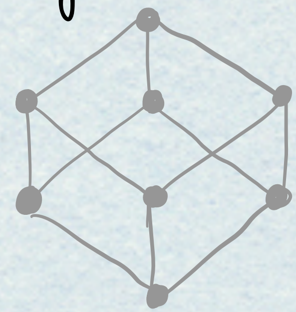

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(c) If $t \not\equiv n \pmod 2$ and $t < n-1$, there are two t-temperate families of maximal size: $\bigcup_{j=0}^{t} \binom{[n]}{\frac{n-t-1}{2} + j}$ and $\bigcup_{j=0}^{t} \binom{[n]}{\frac{n-t+1}{2} + j}$

Partial answer to Question 2

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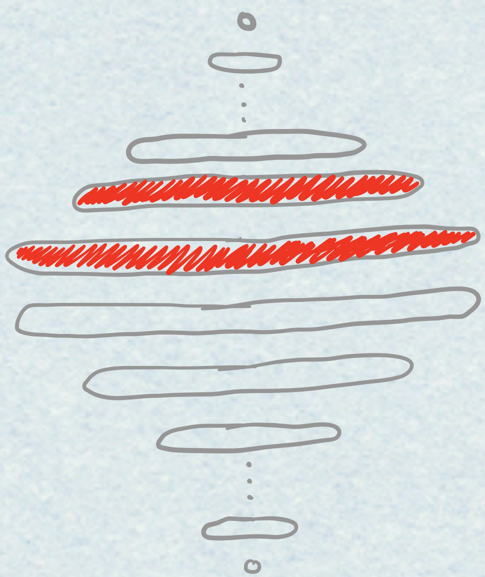

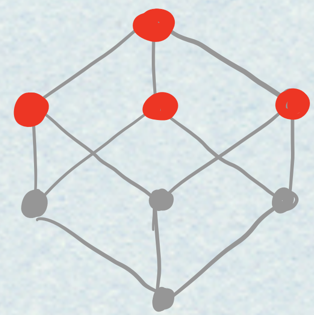

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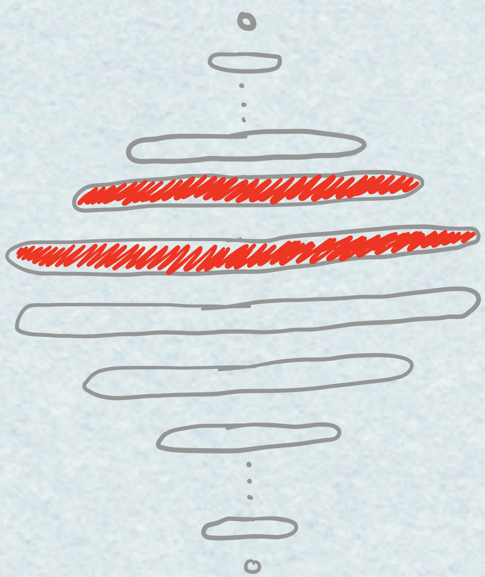

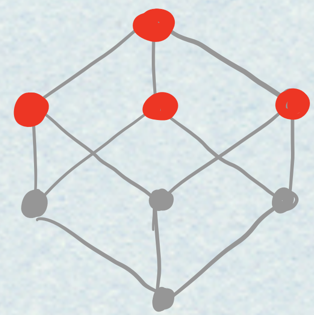

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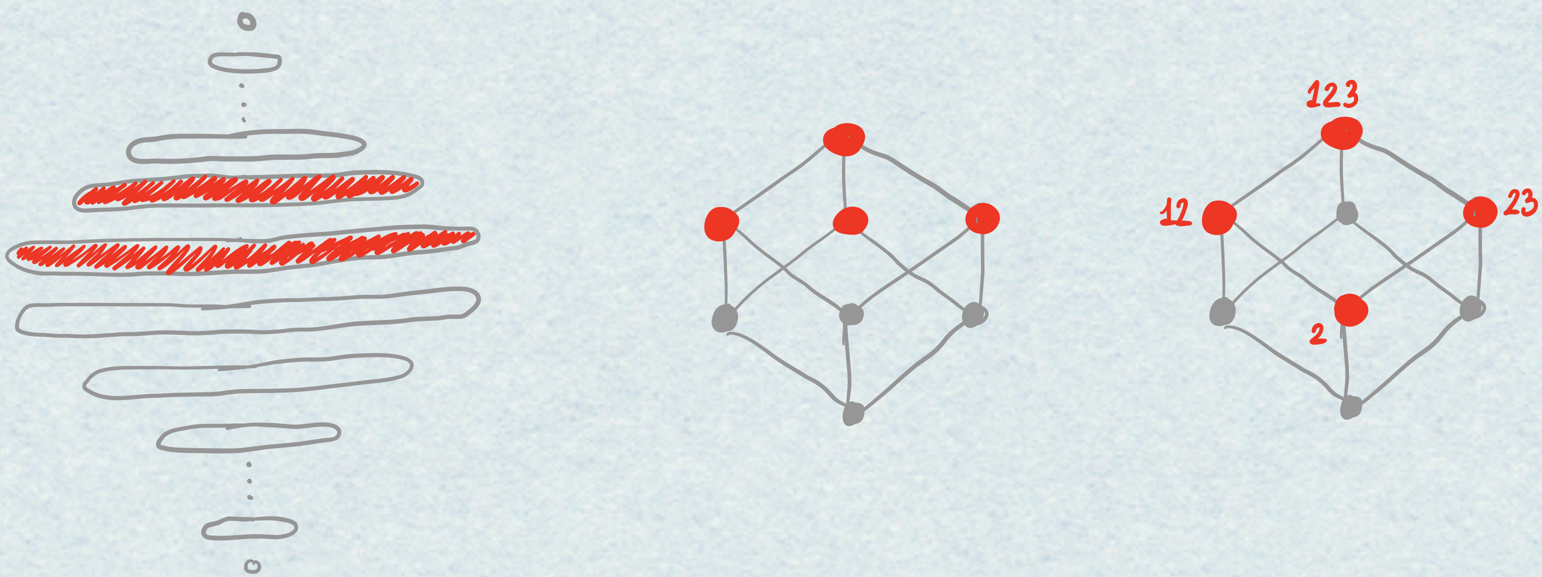

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Conjecture: The maximum size of an intersecting temperate family on $[n]$ for even $n=2k$ is $\frac{1}{2}\binom{2k}{k}+\binom{2k}{k+1}+\binom{2k-1}{k+2}$.

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- all elements of $\binom{[2k]}{k}$ containing 1, and
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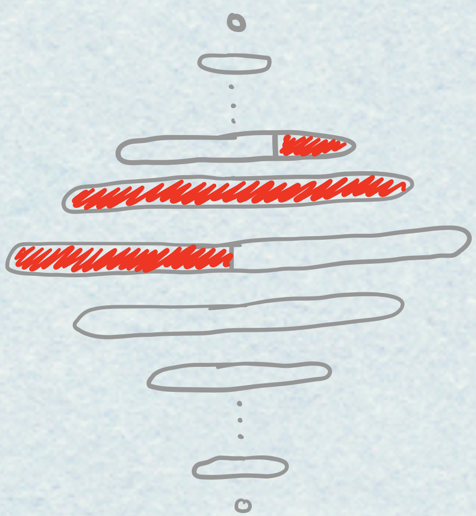

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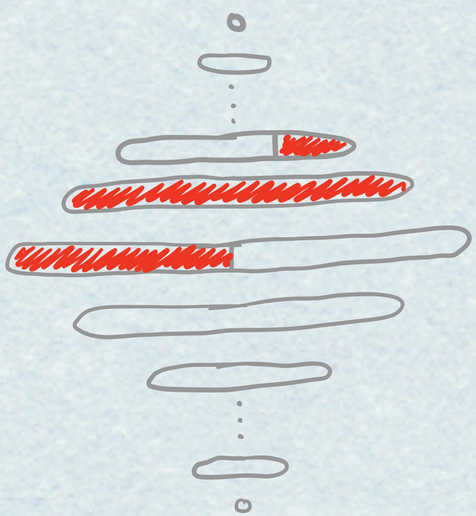

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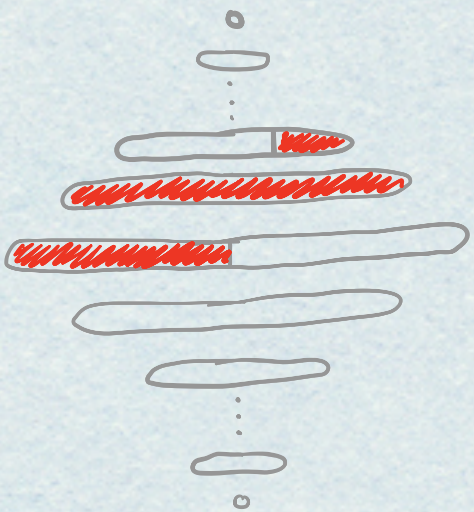

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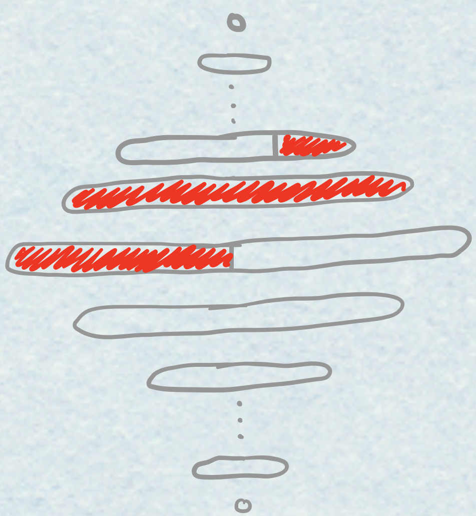

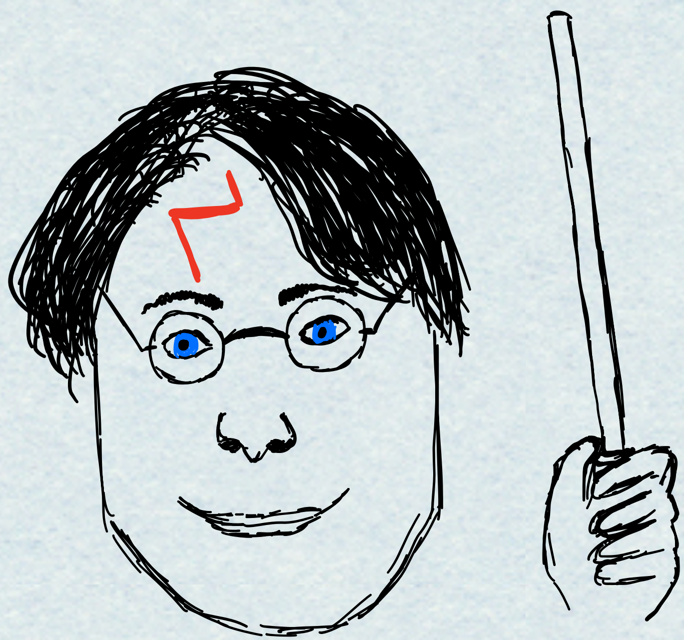

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Pf:
$$\begin{aligned}
\mathbb{E}_{\mathcal{C}}[w(\mathcal{F} \cap \mathcal{C})] &= \sum_{i=0}^{n} w([i]) \cdot \mathbb{P}_{\mathcal{C}}(\mathcal{F}_i \cap \mathcal{C} \neq \emptyset) \\
&= \sum_{i=0}^{n} w([i]) \cdot \frac{|\mathcal{F}_i|}{\binom{n}{i}} \\
&= \sum_{i=0}^{n} |\mathcal{F}_i| = |\mathcal{F}|. \quad \square
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&\quad + \sum_{A \in \mathcal{F}^*} \mathbb{E}_{\mathcal{C}}[w(\mathcal{C} \cap \mathcal{F}) \mid M(\mathcal{C}) = A] \cdot \mathbb{P}_{\mathcal{C}}[M(\mathcal{C}) = A] \\
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&\leq \max_{A \in \mathcal{F}^*} \mathbb{E}_{\mathcal{C}}[w(\mathcal{C} \cap \mathcal{F}) \mid M(\mathcal{C}) = A] \\
&= \max_{A \in \mathcal{F}^*} \sum_{B \in \mathcal{F} \cap \mathcal{P}(A)} w(B) \cdot \mathbb{P}_{\mathcal{C}}[B \in \mathcal{C} \mid M(\mathcal{C}) = A]
\end{aligned}
$$

Proof of Thm 1

$w(A) = \binom{n}{|A|}$, $w(\mathcal{F}) = \sum_{B \in \mathcal{F}} w(B)$

<u>Observation</u>: Let $\mathcal{F}$ be a family on $[n]$ and let $\mathcal{C}$ be a random maximal chain on $[n]$ chosen uniformly at random. Then $\mathbb{E}_{\mathcal{C}}[w(\mathcal{F} \cap \mathcal{C})] = |\mathcal{F}|$.

Denote by $\mathcal{F}^*$ the set of chain-maximal elements of $\mathcal{F}$

there is a maximal chain C on $[n]$ such that the element is maximal in $\mathcal{F} \cap C$

maximal element of $\mathcal{F} \cap \mathcal{C}$

$$
\begin{aligned}
|\mathcal{F}| &= \mathbb{E}_{\mathcal{C}}[w(\mathcal{C} \cap \mathcal{F})] \\
&= \mathbb{E}_{\mathcal{C}}[w(\mathcal{C} \cap \mathcal{F}) \mid \mathcal{C} \cap \mathcal{F} = \emptyset] \cdot \mathbb{P}_{\mathcal{C}}[\mathcal{C} \cap \mathcal{F} = \emptyset] \\
&\quad + \sum_{A \in \mathcal{F}^*} \mathbb{E}_{\mathcal{C}}[w(\mathcal{C} \cap \mathcal{F}) \mid M(\mathcal{C}) = A] \cdot \mathbb{P}_{\mathcal{C}}[M(\mathcal{C}) = A] \\
&= \sum_{A \in \mathcal{F}^*} \mathbb{E}_{\mathcal{C}}[w(\mathcal{C} \cap \mathcal{F}) \mid M(\mathcal{C}) = A] \cdot \mathbb{P}_{\mathcal{C}}[M(\mathcal{C}) = A] \\
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&= \max_{A \in \mathcal{F}^*} \sum_{B \in \mathcal{F} \cap \mathcal{P}(A)} w(B) \cdot \mathbb{P}_{\mathcal{C}}[B \in \mathcal{C} \mid M(\mathcal{C}) = A] \\
&= \max_{A \in \mathcal{F}^*} \sum_{B \in \mathcal{F} \cap \mathcal{P}(A)} \binom{n}{|B|} \Big/ \binom{|A|}{|B|}
\end{aligned}
$$

Proof of Thm 1

$$
\begin{aligned}
|\mathcal{F}| &= \mathbb{E}_{\mathcal{C}}\left[w(\mathcal{C} \cap \mathcal{F})\right] \\
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&\qquad + \sum_{A \in \mathcal{F}^*} \mathbb{E}_{\mathcal{C}}\left[w(\mathcal{C} \cap \mathcal{F}) \mid M(\mathcal{C}) = A\right] \cdot \mathbb{P}_{\mathcal{C}}\left[M(\mathcal{C}) = A\right] \\
&= \sum_{A \in \mathcal{F}^*} \mathbb{E}_{\mathcal{C}}\left[w(\mathcal{C} \cap \mathcal{F}) \mid M(\mathcal{C}) = A\right] \cdot \mathbb{P}_{\mathcal{C}}\left[M(\mathcal{C}) = A\right] \\
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&\qquad + \sum_{A\in\mathcal{F}^*} \mathbb{E}_{\mathcal{C}}\left[w(\mathcal{C}\cap\mathcal{F}) \mid M(\mathcal{C})=A\right]\cdot \mathbb{P}_{\mathcal{C}}\left[M(\mathcal{C})=A\right] \\
&= \sum_{A\in\mathcal{F}^*} \mathbb{E}_{\mathcal{C}}\left[w(\mathcal{C}\cap\mathcal{F}) \mid M(\mathcal{C})=A\right]\cdot \mathbb{P}_{\mathcal{C}}\left[M(\mathcal{C})=A\right] \\
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&= \max_{A\in\mathcal{F}^*} \sum_{B\in\mathcal{F}\cap\mathcal{P}(A)} \underbrace{\binom{n}{|B|}\Big/\binom{|A|}{|B|}}_{=\binom{n}{|A|}\binom{n-|B|}{n-|A|}}
\end{aligned}
$$

non-decreasing in $|B|$

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&= \max_{A\in\mathcal{F}^*} \sum_{B\in\mathcal{F}\cap\mathcal{P}(A)} \underbrace{\binom{n}{|B|}\Big/\binom{|A|}{|B|}}_{=\binom{n}{|A|}\binom{n-|B|}{n-|A|}} \qquad \text{non-decreasing in } |B| \\
&\leqslant \max_{A\in\mathcal{F}^*} \sum_{j=\max(0,\,|A|-t)}^{|A|} \binom{n}{j}
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&\leq \sum_{j=0}^{t} \binom{n}{\lfloor \frac{n-t}{2} \rfloor + j} \qquad \square
\end{aligned}
$$

Proof of Thm 3

Thm 3: The maximum size of an intersecting temperate family on $[n]$ for odd $n=2k-1$ is $\binom{2k-1}{k}+\binom{2k-1}{k+1}$.

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$$\geq \frac{|\mathcal{F}_{k-1}|+|\mathcal{F}_k|}{\binom{2k-1}{k}} + \frac{1}{\binom{2k-1}{k+1}} \sum_{\substack{0\le i\le n\\ i\ne k-1,k}} |\mathcal{F}_i|$$

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&= \frac{|\mathcal{F}_{k-1}|+|\mathcal{F}_k|}{\binom{2k-1}{k}} + \frac{|\mathcal{F}|-(|\mathcal{F}_{k-1}|+|\mathcal{F}_k|)}{\binom{2k-1}{k+1}}
\end{aligned}
$$

After rearranging: $|\mathcal{F}|\le 2\binom{2k-1}{k}+(|\mathcal{F}_{k-1}|+|\mathcal{F}_k|)\left(1-\frac{\binom{2k-1}{k+1}}{\binom{2k-1}{k}}\right)$

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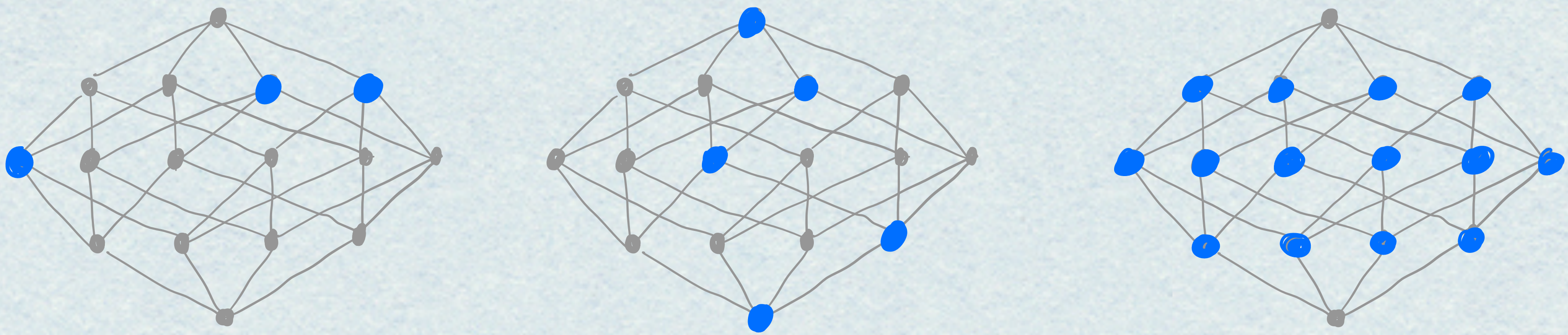
As $\mathcal{F}$ is intersecting, $|\mathcal{F}_{k-1}|+|\mathcal{F}_k|\leq\binom{2k-1}{k}$. The result follows. $\square$



Thank you for your attention!